Filter

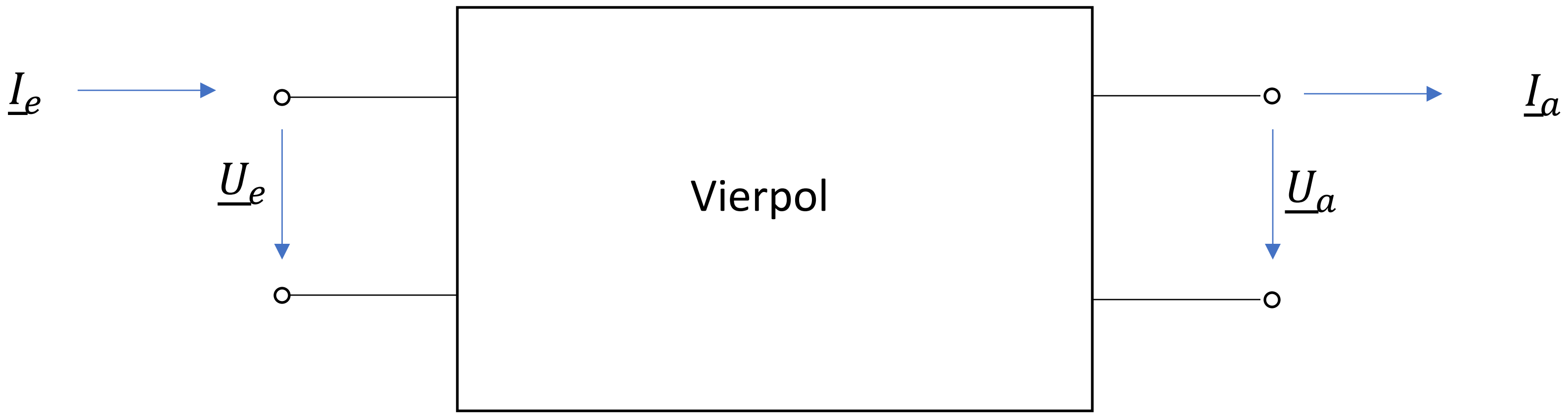
IGA e.V.
Interessengemeinschaft Amateurfunk Pulheim/Rhein-Erft

DARC
Deutscher Amateur-Radio-Club e.V.
Bundesverband für Amateurfunk in Deutschland

Ortsverband Pulheim G40

Filter, Einführung

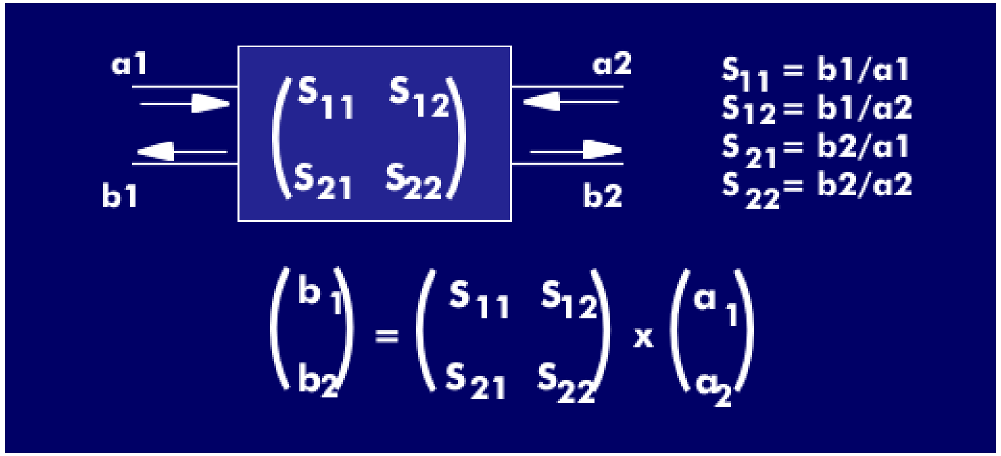
Vierpol

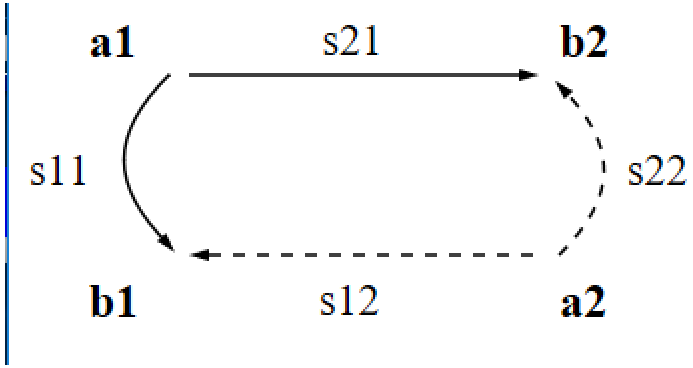


Übertragungsverhalten bei $\underline{I}_a = 0$ ist $A(j\omega)$

$$A(j\omega) = \frac{\underline{U}_a}{\underline{U}_e}$$

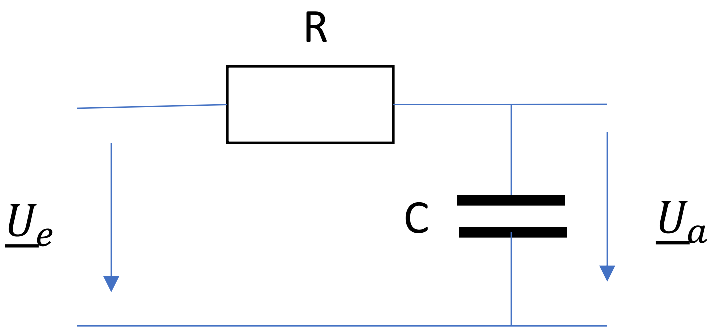
Streuparameter





Eingangsreflektionsfaktor S_{11}
Ausgangsreflektionsfaktor S_{22}
Einfügedämpfung S_{21}
Rückwärtsdämpfung S_{12}

Tiefpass 1. Ordnung

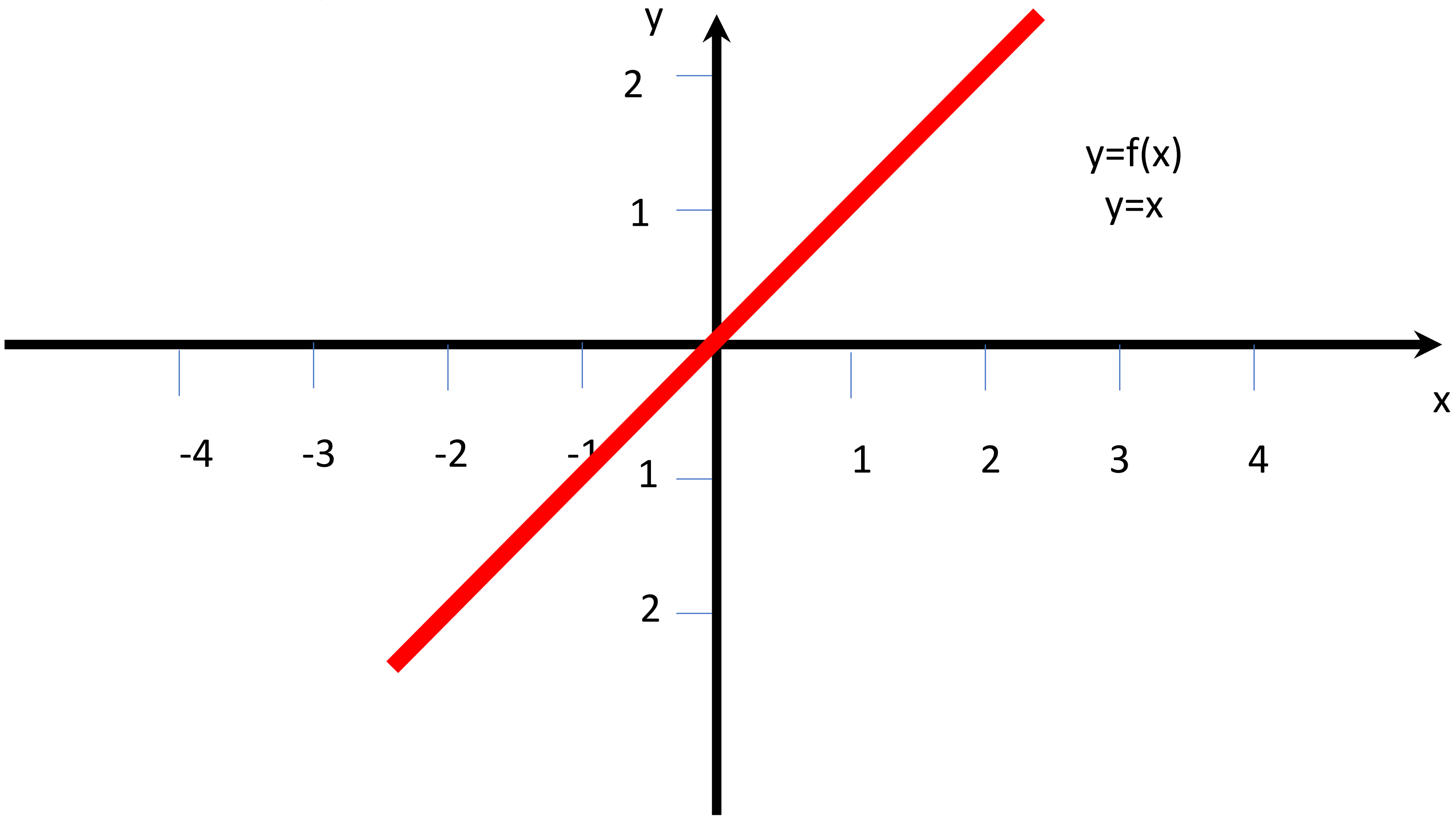


$$\frac{\underline{U}_e}{\underline{U}_a} = \frac{R + X_c}{X_c}$$

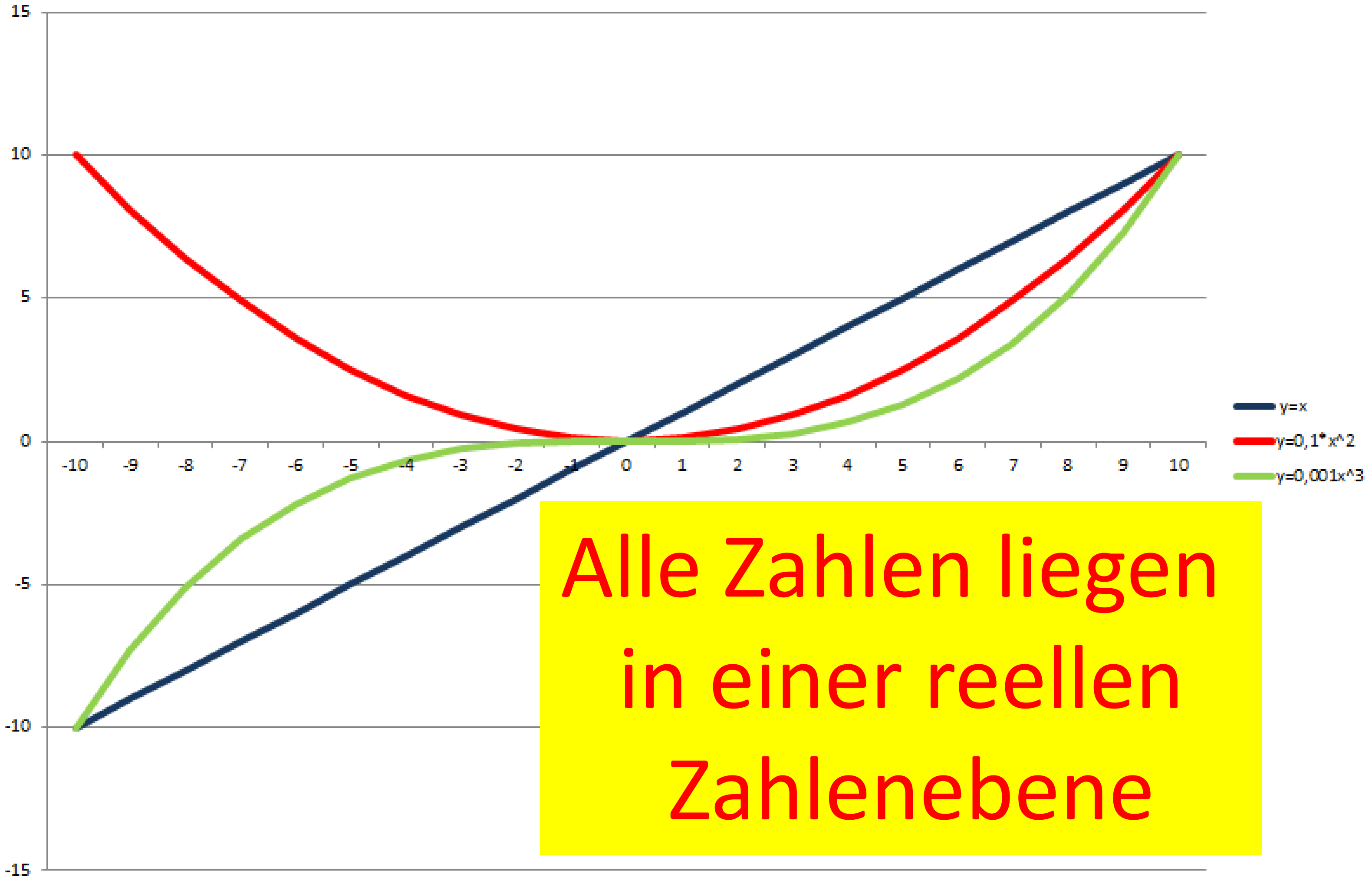
Da die Phasenlage des Stromes bei reinen Widerstandsnetzwerken anders ist,
als bei Netzwerken mit Energiespeichern, wie Induktivitäten und Kapazitäten,
muss man bei der mathematischen Behandlung solcher Systeme diese Besonderheiten berücksichtigen

Komplexe Größen

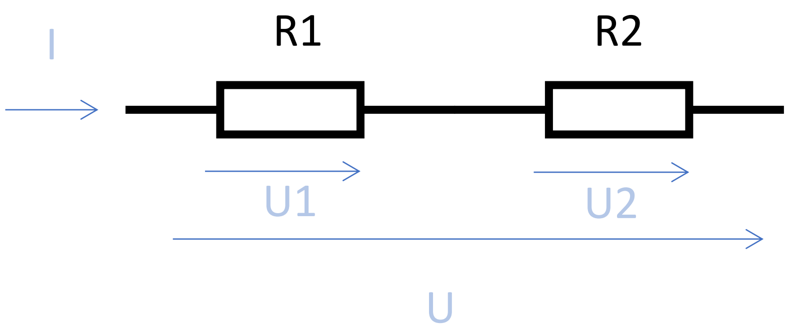
Darstellung im kartesischen Koordinatensystem



Darstellung im kartesischen Koordinatensystem



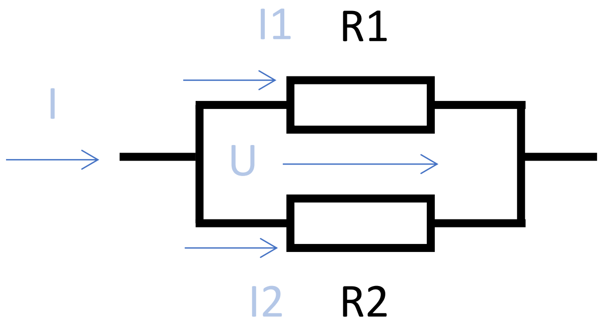
Widerstand



R = R1 + R2 ; gleichphasig

U = U1 + U2 ; $I = \frac{U}{R}$; gleichphasig

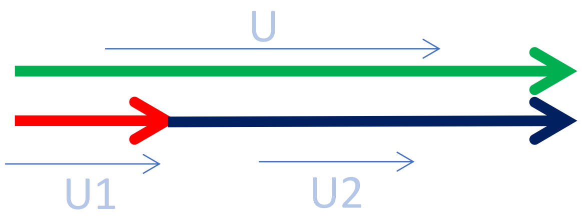
Beispiel: Spannungsteiler $\frac{U1}{U2} = \frac{R1}{R2}$



$$R = \frac{R1 * R2}{(R1 + R2)}$$

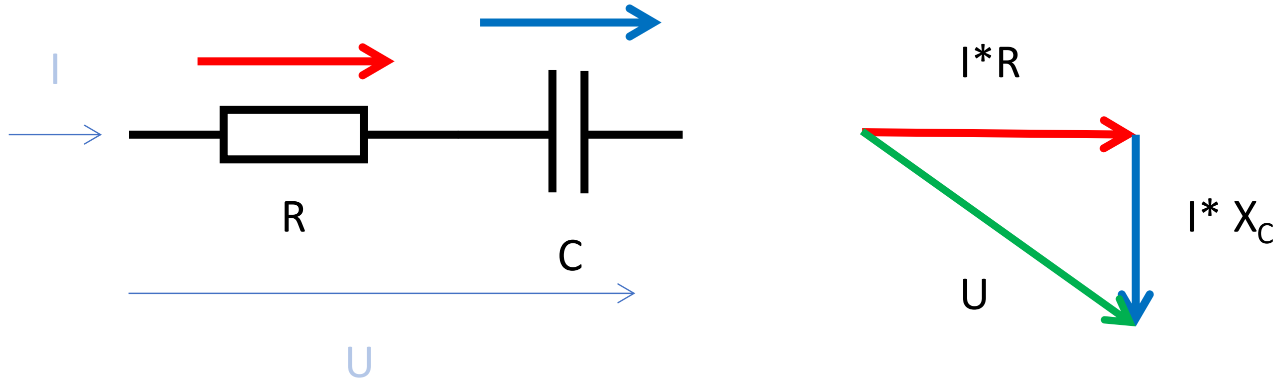
I = I1 + I2 ; $I = \frac{U}{R}$; gleichphasig

$$\frac{I1}{I2} = \frac{R2}{R1}$$



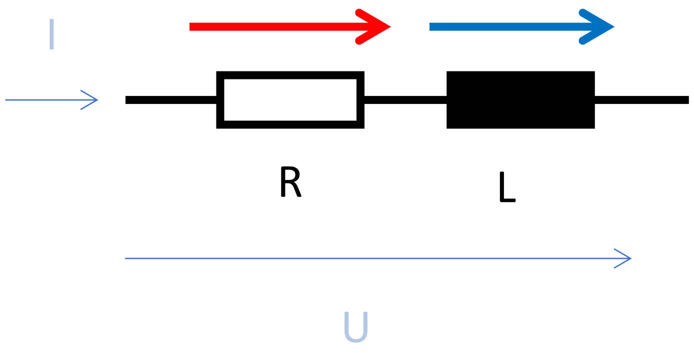
Kondensator

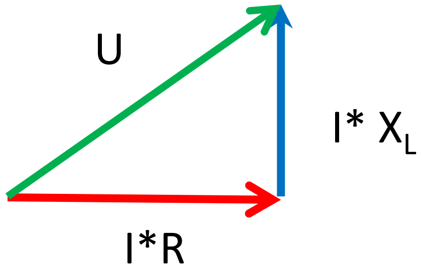
$X_C = \frac{U}{I} = \frac{1}{2\pi f C} = \frac{1}{\omega C}$ mit $\varphi = -90°$ (Def.: $\varphi = \varphi_u - \varphi_i$)



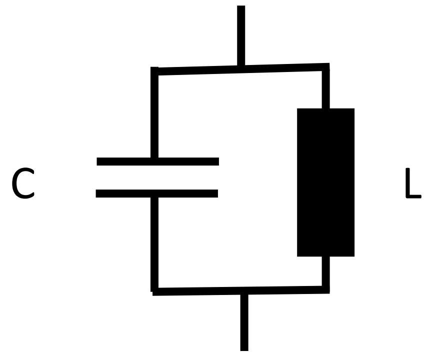
Induktivität

$X_L = \frac{U}{I} = 2\pi f L = \omega L$ mit $\varphi=90°$ (Def.: $\varphi = \varphi_u - \varphi_i$)





Zusammenschaltung Kapazität und Induktivität



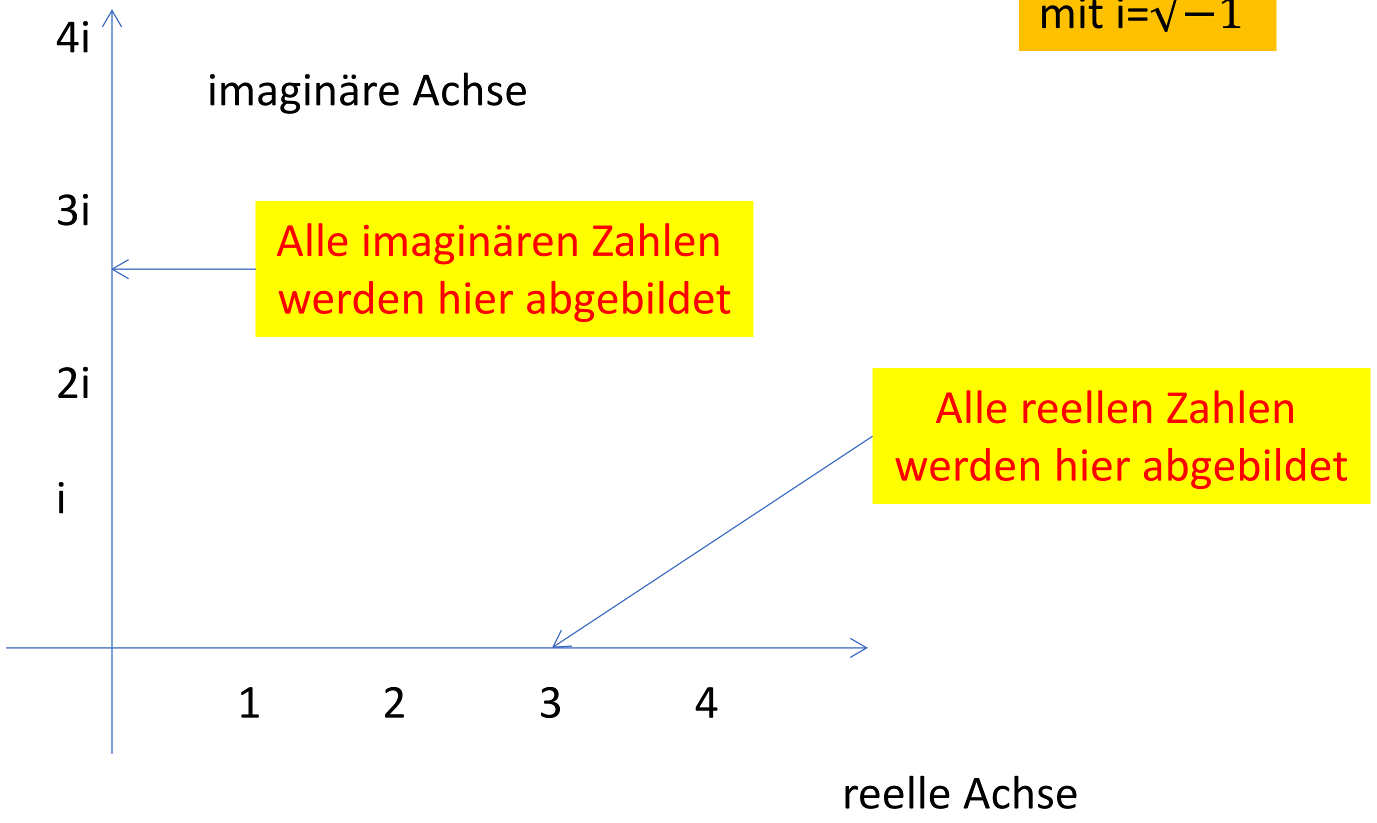
$$\boxed{Z = \frac{X_C * X_L}{X_C + X_L}}$$

?

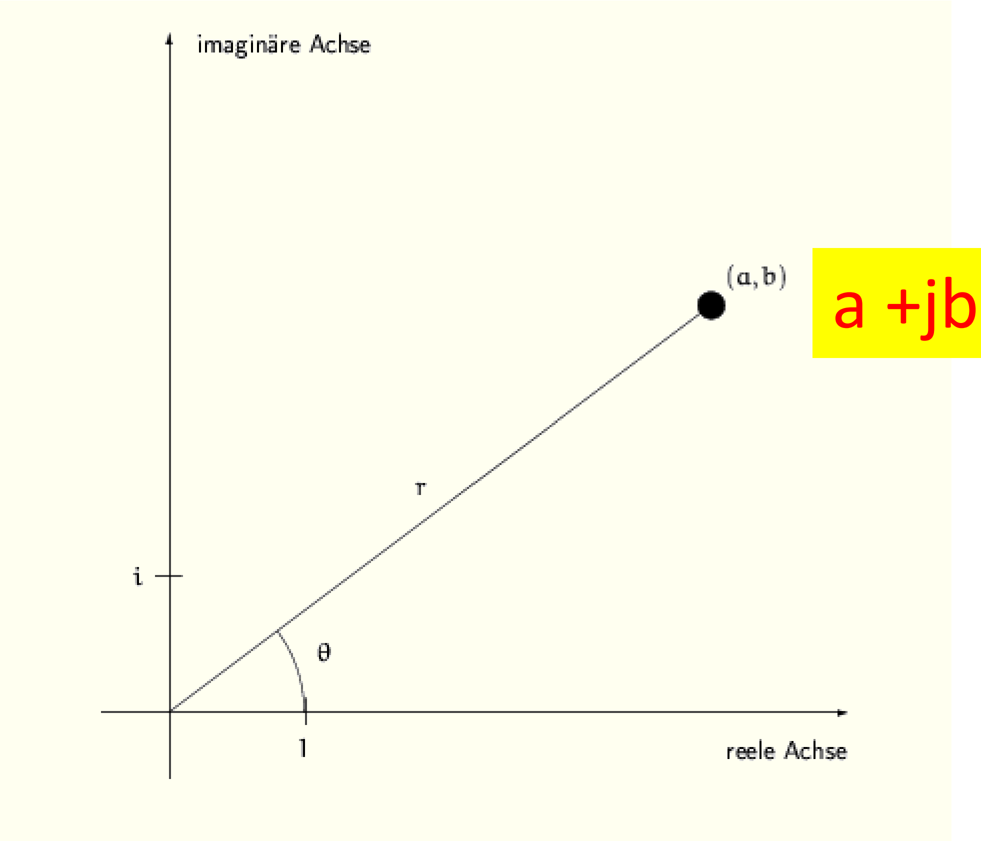
Wie berücksichtigt man nun einfach die unterschiedlichen Phasenlagen?

Gaußsche Zahlenebene

mit $i=\sqrt{-1}$



Gaußsche Zahlenebene



a +jb

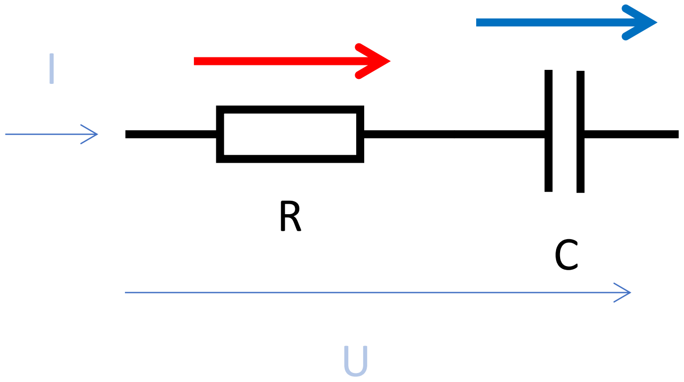
mit i=$\sqrt{-1}$

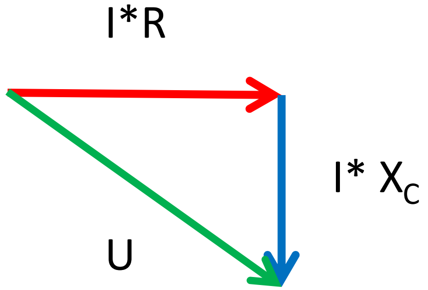
Definition

$$\boxed{X_C = \frac{1}{j\omega C}}$$

mit $\frac{1}{j} = -j$

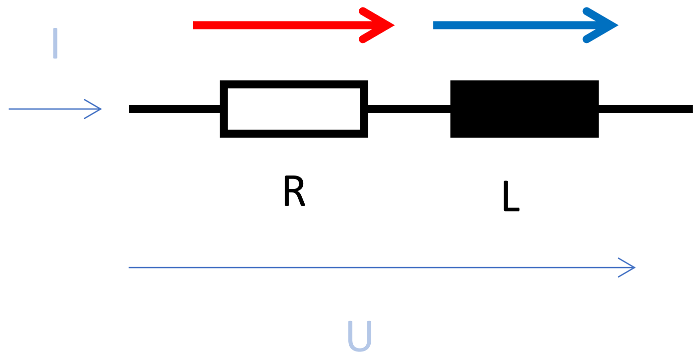
$$\boxed{X_C = -\mathrm{j}\frac{1}{\omega C}}$$

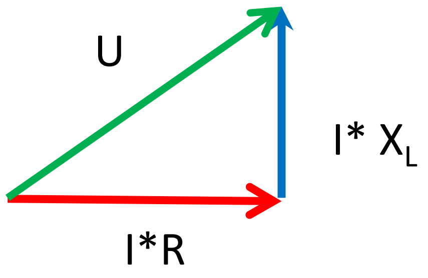




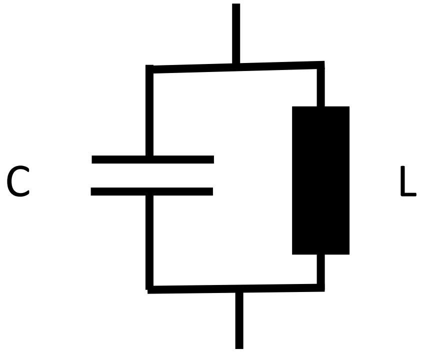
Definition

$$X_L = j\omega L$$





Zusammenschaltung Kapazität und Induktivität



$$\boxed{Z = \frac{X_C * X_L}{X_C + X_L}}$$

Wie berücksichtigt man nun einfach die unterschiedlichen Phasenlagen?

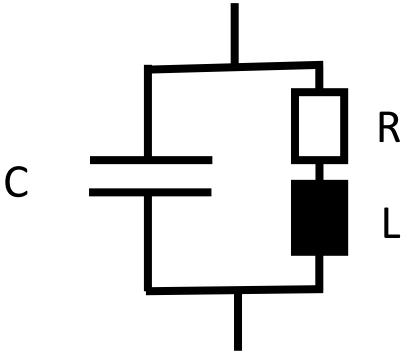
Zusammenschaltung Kapazität und Induktivität

$$\underline{Z} = \frac{(R + j\omega L))\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}$$

$$(a-b)(a+b) = a^2 - b^2$$

$$jj = -1$$

$$\frac{1}{j} = -j$$



$$\underline{Z} = \frac{\underline{X}_c(R + \underline{X}_L)}{\underline{X}_c + \underline{X}_L + R}$$

$$\underline{Z} = \frac{\frac{R}{j\omega C} + \frac{L}{C}}{R + j(\omega L - \frac{1}{\omega C})} = \frac{(\frac{L}{C} - j\frac{R}{\omega C})(R - j(\omega L - \frac{1}{\omega C})}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

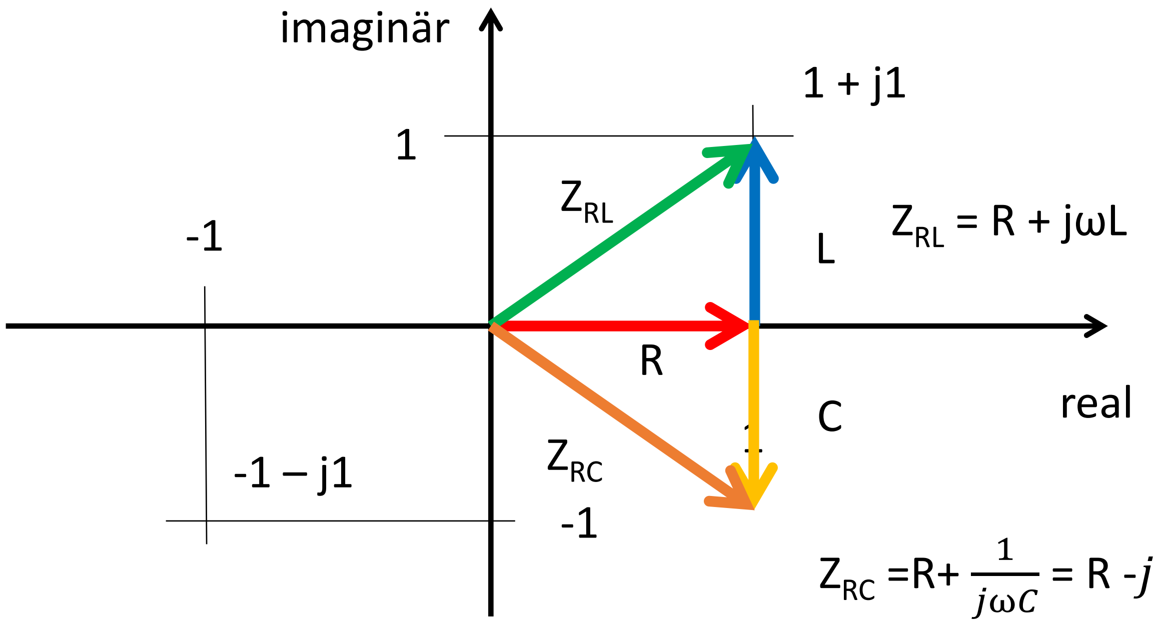
Re

Im

$$\underline{Z} = \frac{R\frac{L}{C} - j(\left(\omega\frac{L^2}{C} - \frac{L}{\omega C^2}\right) - j\frac{R^2}{\omega C} - (\frac{RL}{C} - \frac{R}{(\omega C)^2}}{R^2 + (\omega L - \frac{1}{\omega C})^2} = \frac{\frac{R}{(\omega C)^2} - j(\omega\frac{L^2}{C} - \frac{L}{\omega C^2} + \frac{R^2}{\omega C})}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

mit der Gaußschen Zahlenebene

für sinusförmige Verhältnisse



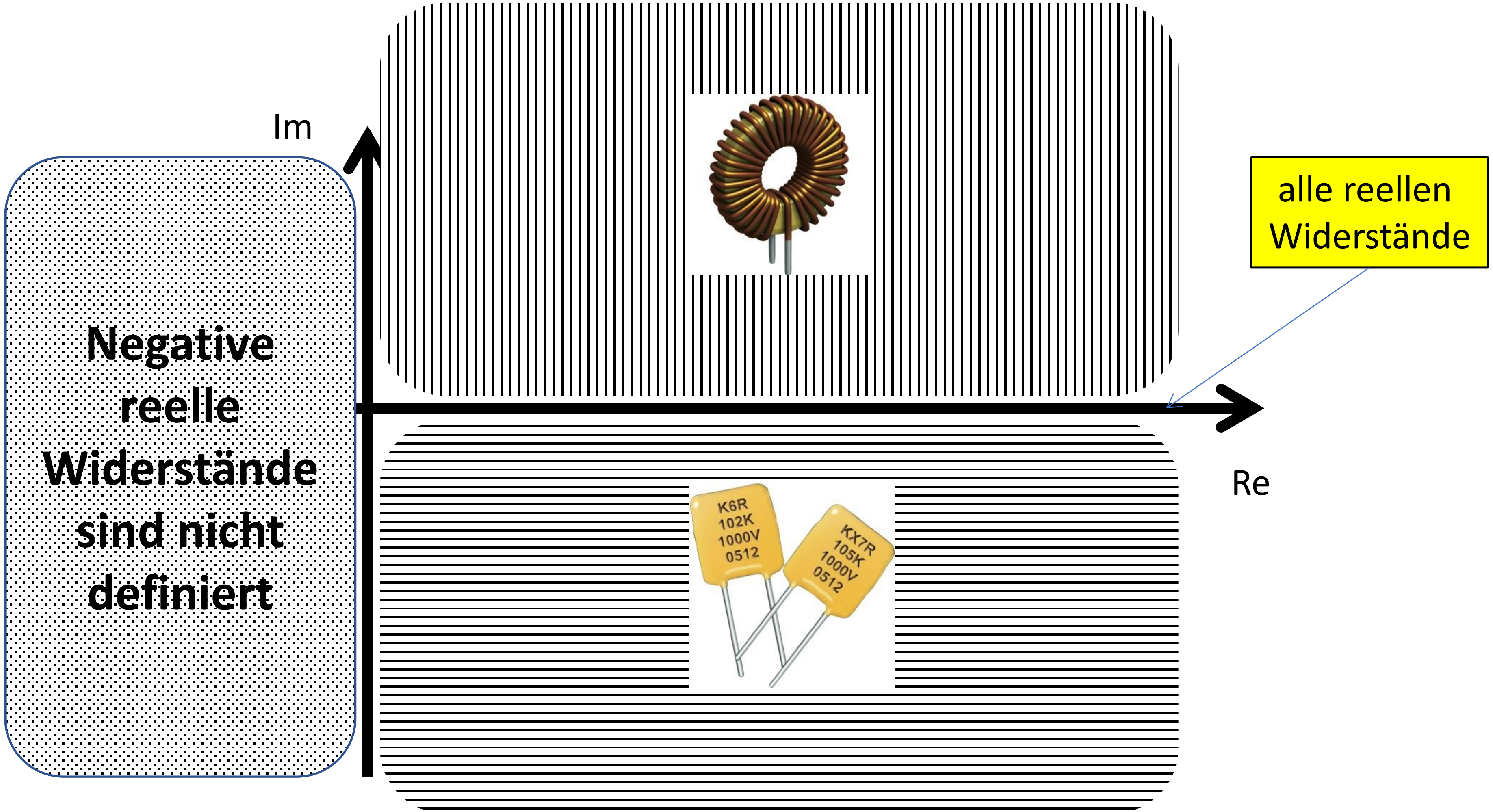
Z_{RL} = R + jωL

$Z_{RC} = R + \frac{1}{j\omega C} = R - j\,\frac{1}{\omega C}$

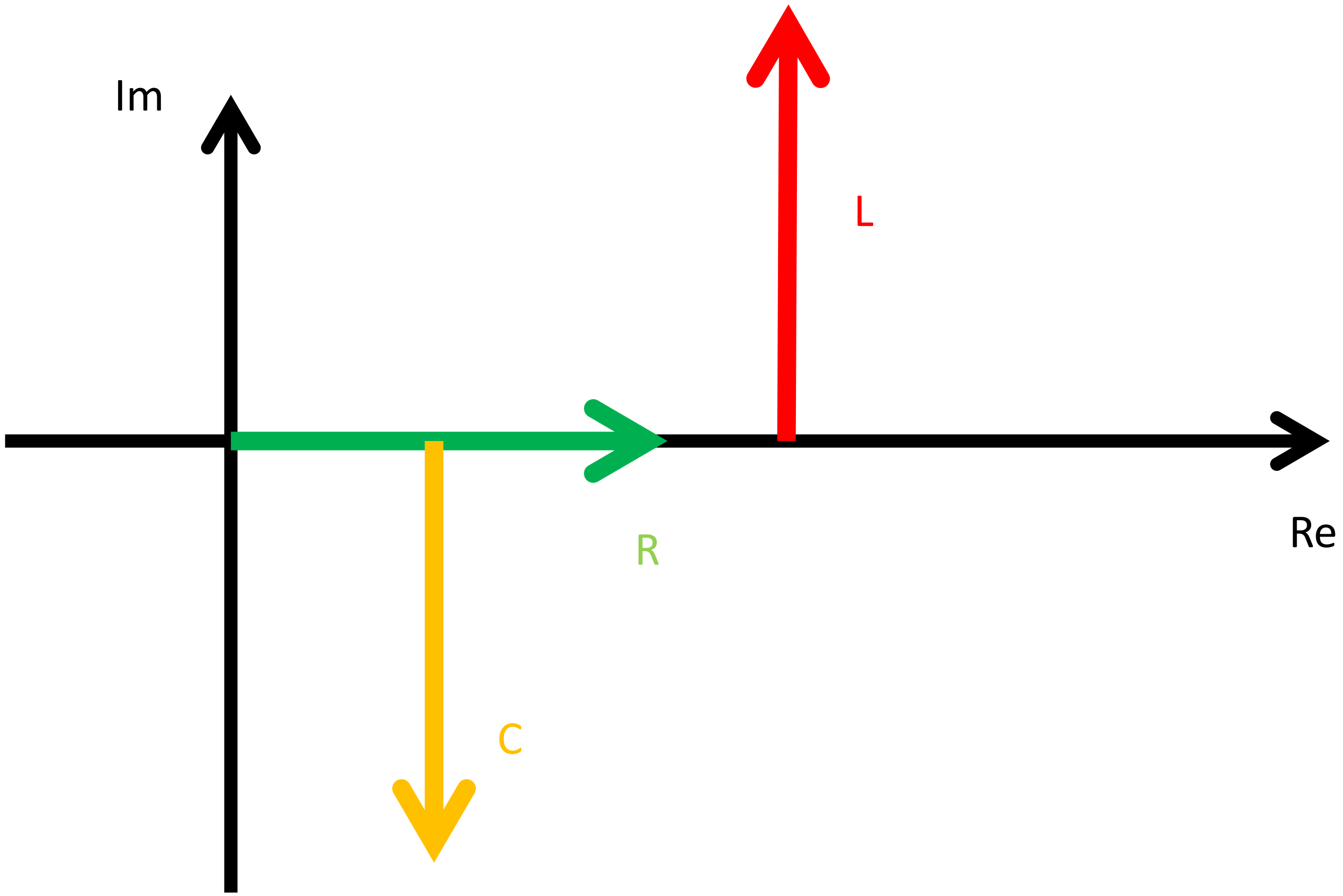
$Z_{RL} + Z_{RC} = R + j\omega L + R - j\,\frac{1}{\omega C}$

Damit ist die phasenrichtige Berechnung einfach möglich

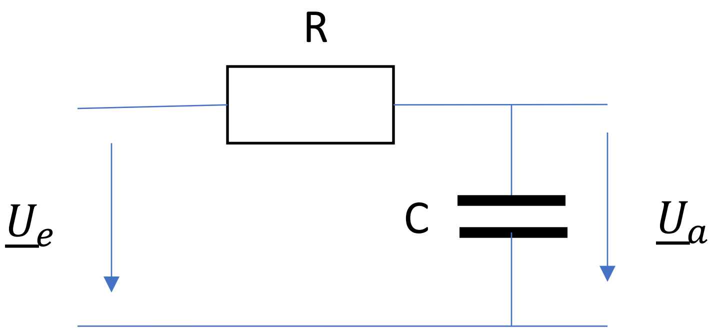
Gaußsche Zahlenebene



Gaußsche Zahlenebene

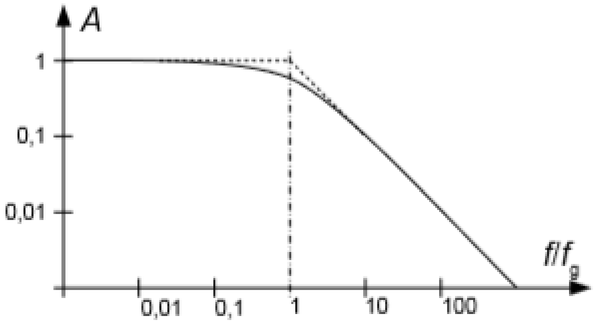


Tiefpass 1. Ordnung

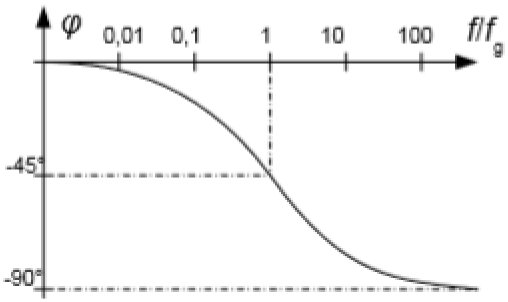


$$\frac{\underline{U_e}}{\underline{U_a}} = \frac{R + \frac{1}{j\omega C}}{\frac{1}{j\omega C}} = 1 + j\omega RC \qquad \underline{A}_T = \frac{1}{1 + j\omega RC}$$

$$\left|\underline{A}_T(\omega)\right| = A(\omega) = \frac{1}{\sqrt{1 + (\omega RC)^2}} \qquad \varphi(\omega) = -\arctan(\omega RC)$$

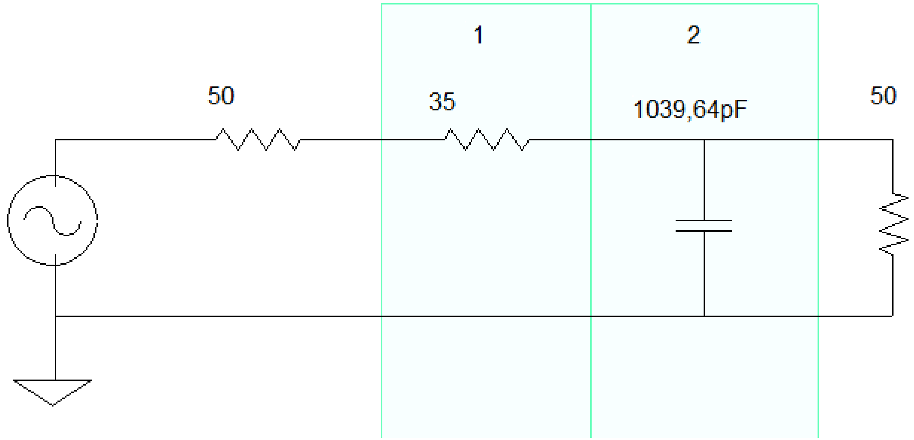


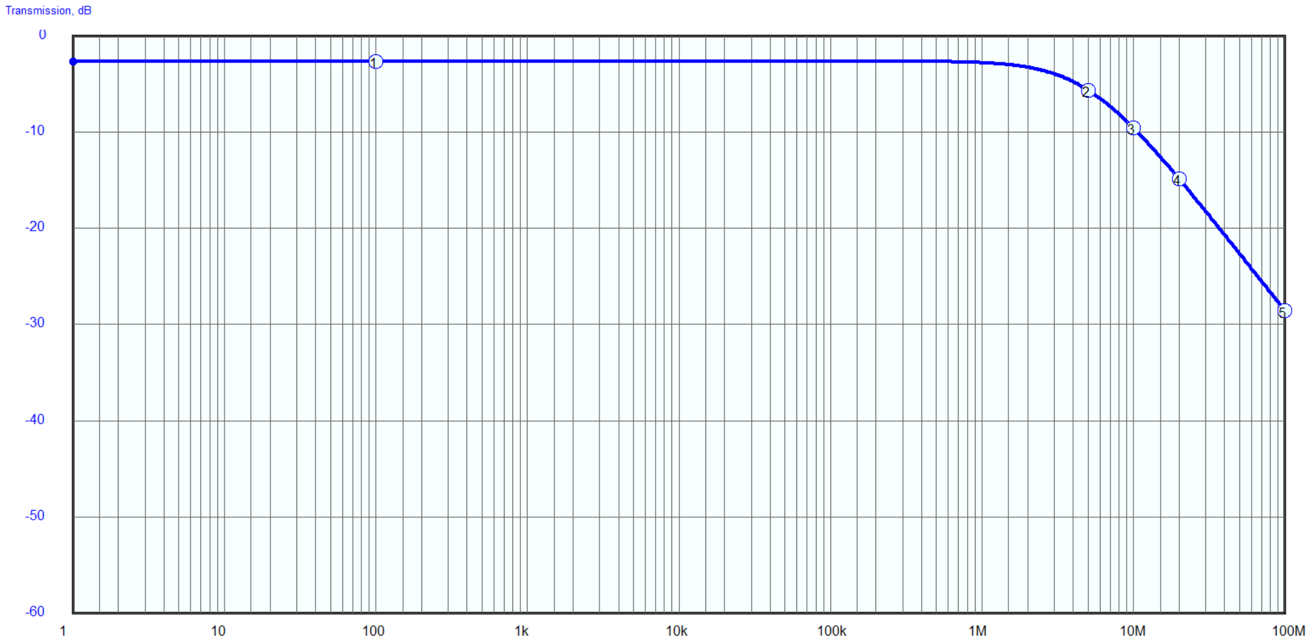
: Amplitudenfrequenzgang beim Tiefpass 1. Ordnung



: Phasenfrequenzgang beim Tiefpass 1. Ordnung

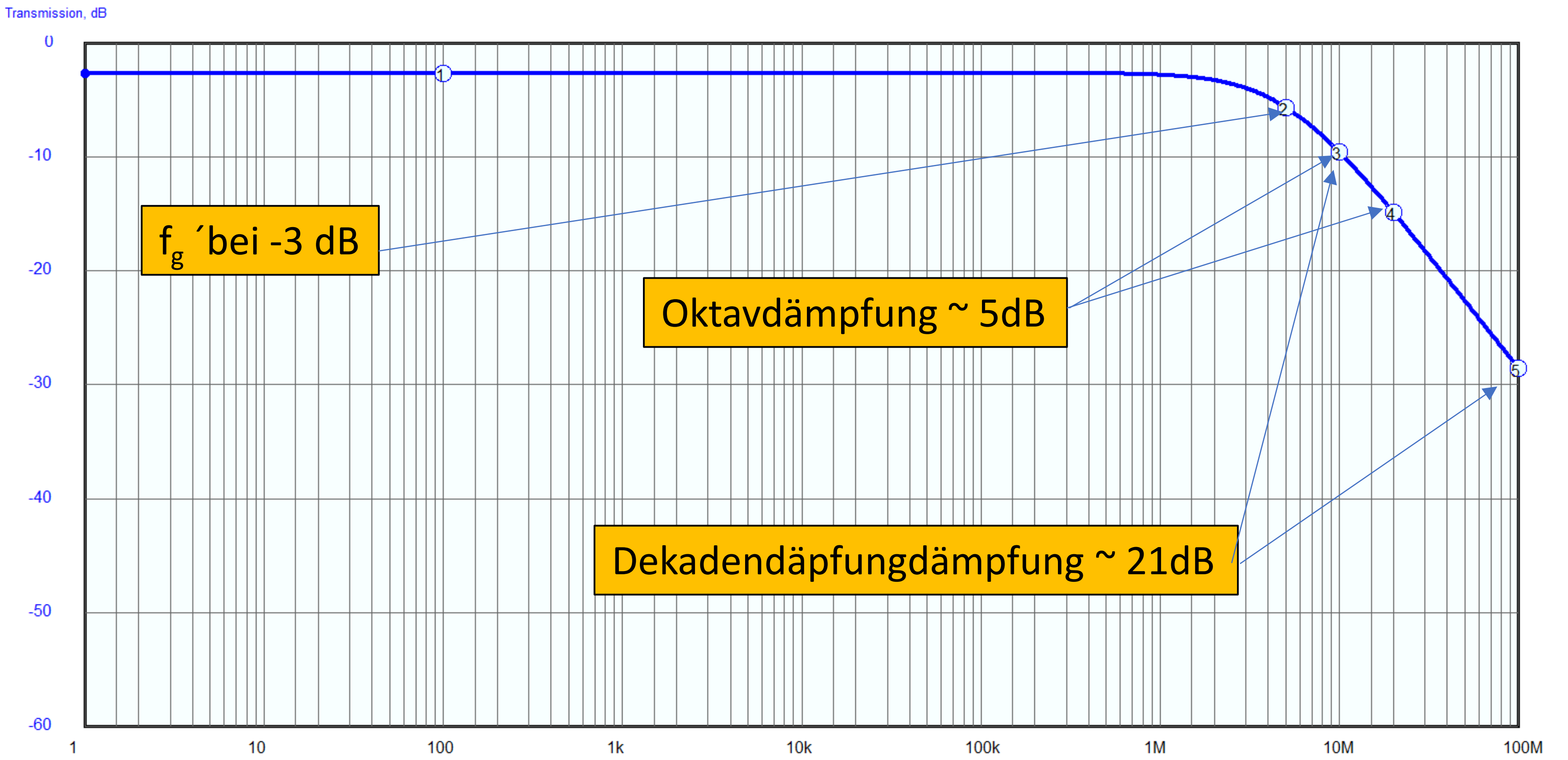
Beispiel (ELSIE)





	Freq.:	Trans.:	Angle:	Return:	Delay:	VSWR:	Zmag.:	Zangle:	Real:	Imag.:
1	100	-2,6067	-0,00113	-11,725	31,48n	1,7	85	-0,00106	85	-0,00157
2	5,058M	-5,6213	-45	-13,087	15,87n	1,5695	54,103	-24,608	49,189	-22,529
3	10M	-9,5216	-63,158	-14,151	6,5n	1,4878	42,159	-20,041	39,606	-14,447
4	20M	-14,822	-75,787	-14,777	1,924n	1,4463	37,06	-12,086	36,239	-7,7599
5	100M	-28,543	-87,077	-15,054	81,74p	1,4293	35,087	-2,5971	35,051	-1,5899

Beispiel (ELSIE)



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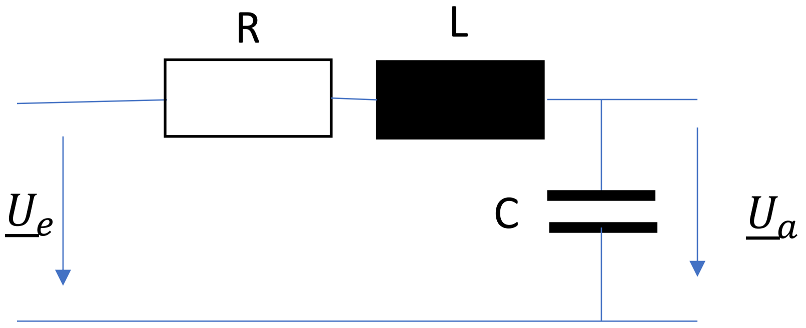
Tiefpass 2. Ordnung



Die Ordnung beschreibt die Potenz
der Frequenz im Nenner. Hier 2.
Je höher die Potenz, desto steiler der Abfall

$$\frac{\underline{U}_e}{\underline{U}_a} = \frac{R + j\omega L + \frac{1}{j\omega C}}{\frac{1}{j\omega C}} = 1 - \omega^2 LC + j\omega RC$$

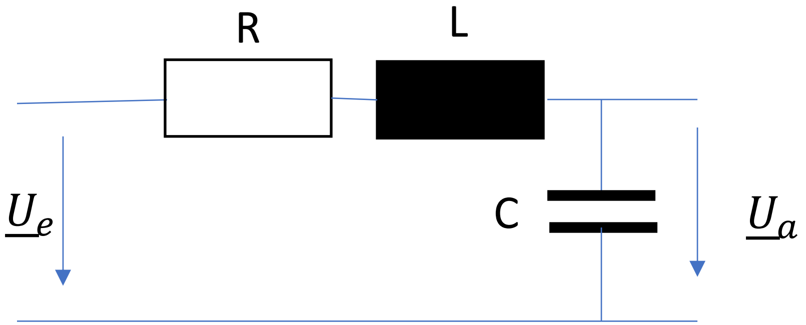
$$\underline{A}_T(\omega) = \frac{1}{1 - \omega^2 LC + j\omega RC}$$

Serienschwingkreis, der durch R bedämpft wird. L und C bestimmen die Eckfrequenz

$\omega_g = \omega_0 = \frac{1}{\sqrt{LC}}$ Knickfrequenz $\quad D = R\sqrt{\frac{C}{L}}$ Dämpfung

$$\underline{A}_T(\omega) = \frac{1}{1 - (\frac{\omega}{\omega_0})^2 + jD\frac{\omega}{\omega_0}}$$

Tiefpass 2. Ordnung



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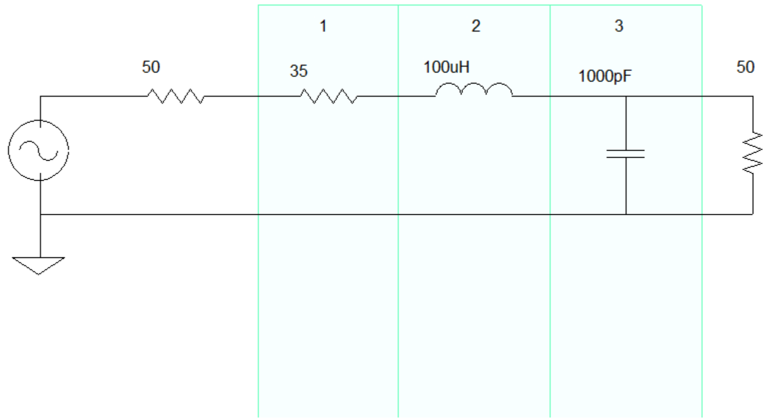
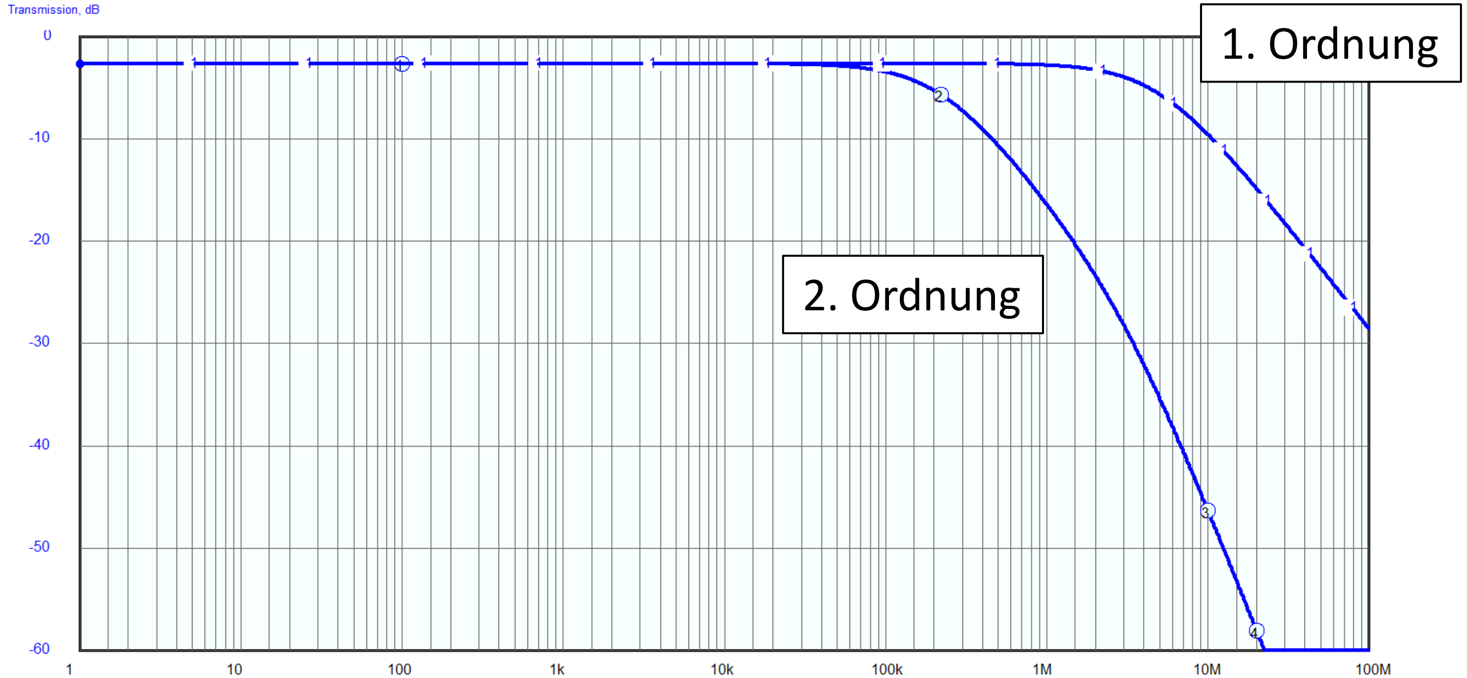
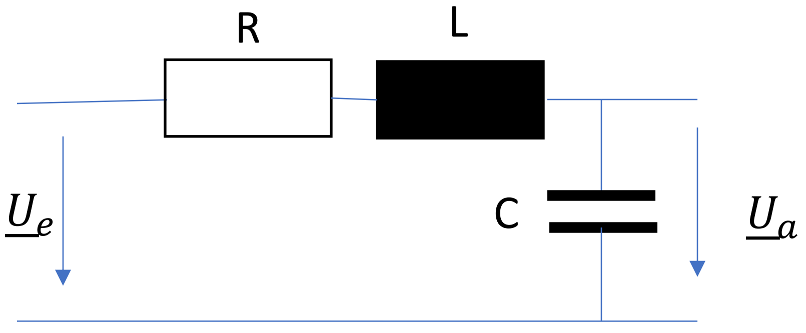
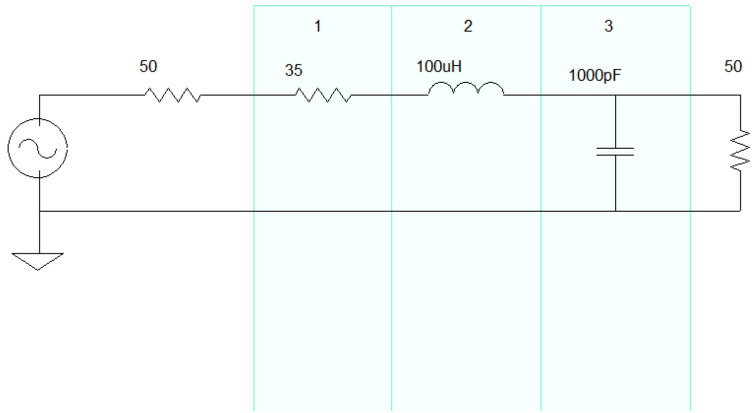


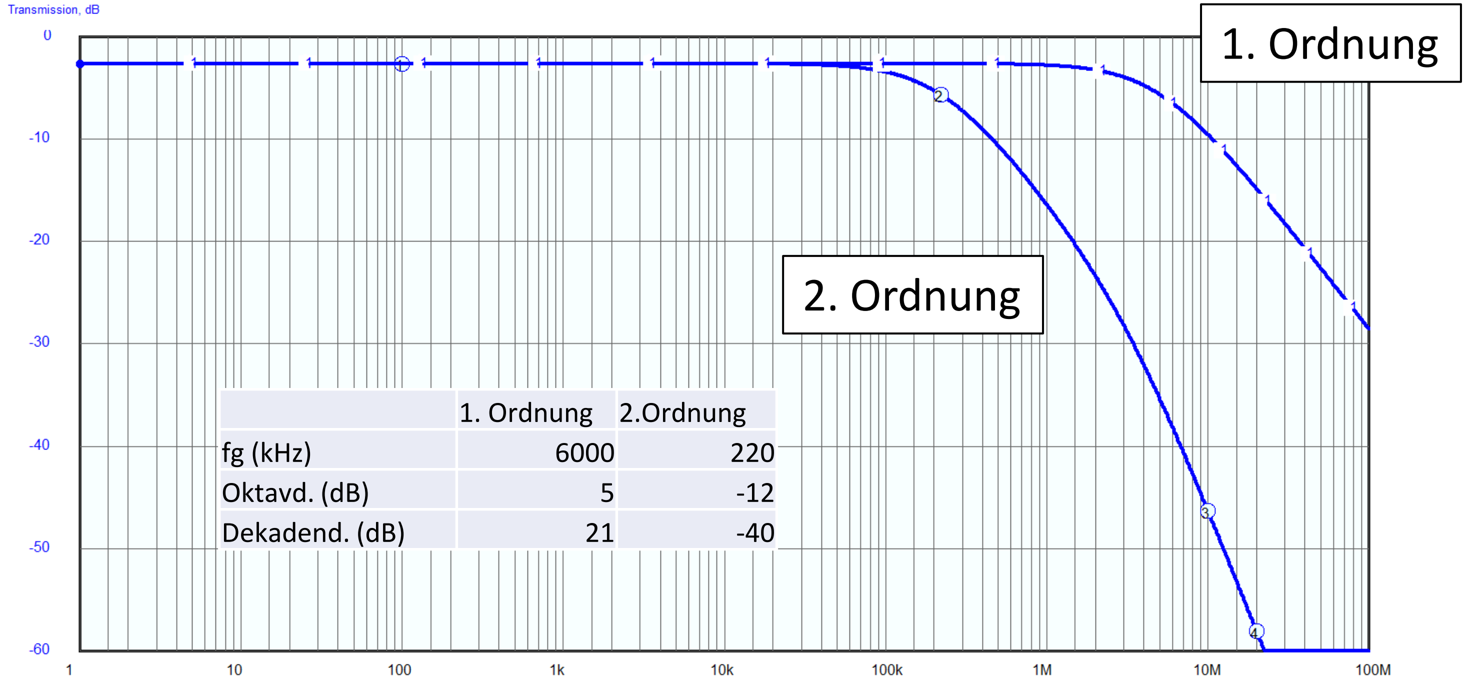


	Freq.:	Trans.:	Angle:	Return:	Delay:	VSWR:	Zmag.:	Zangle:	Real:	Imag.:
1	100	-2,6168	-0,02777	-11,697	771,4n	1,7031	85,157	0,04122	85,157	0,06126
2	220k	-5,6286	-48,927	-2,7323	416,2n	6,4103	159,31	57,789	84,918	134,79
3	10M	-46,308	-161,5	-0,00088	4,931n	>1000	6,269k	89,637	39,763	6,269k
4	20M	-58,052	-170,53	-0,0002	1,315n	>1000	12,559k	89,834	36,396	12,559k
5	100M	-85,911	-178,07	0.0000	53,75p	>1000	62,83k	89,968	35,206	62,83k

Tiefpass 2. Ordnung







	1. Ordnung	2.Ordnung
fg (kHz)	6000	220
Oktavd. (dB)	5	-12
Dekadend. (dB)	21	-40

	Freq.:	Trans.:	Angle:	Return:	Delay:	VSWR:	Zmag.:	Zangle:	Real:	Imag.:
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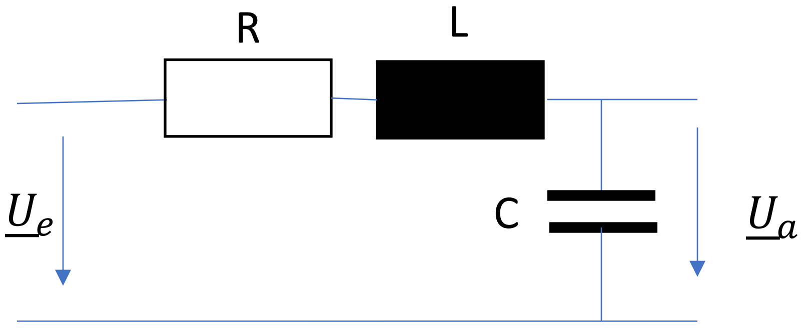
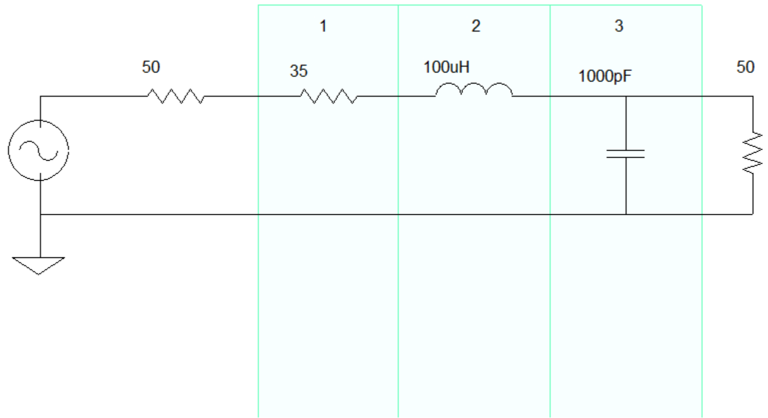
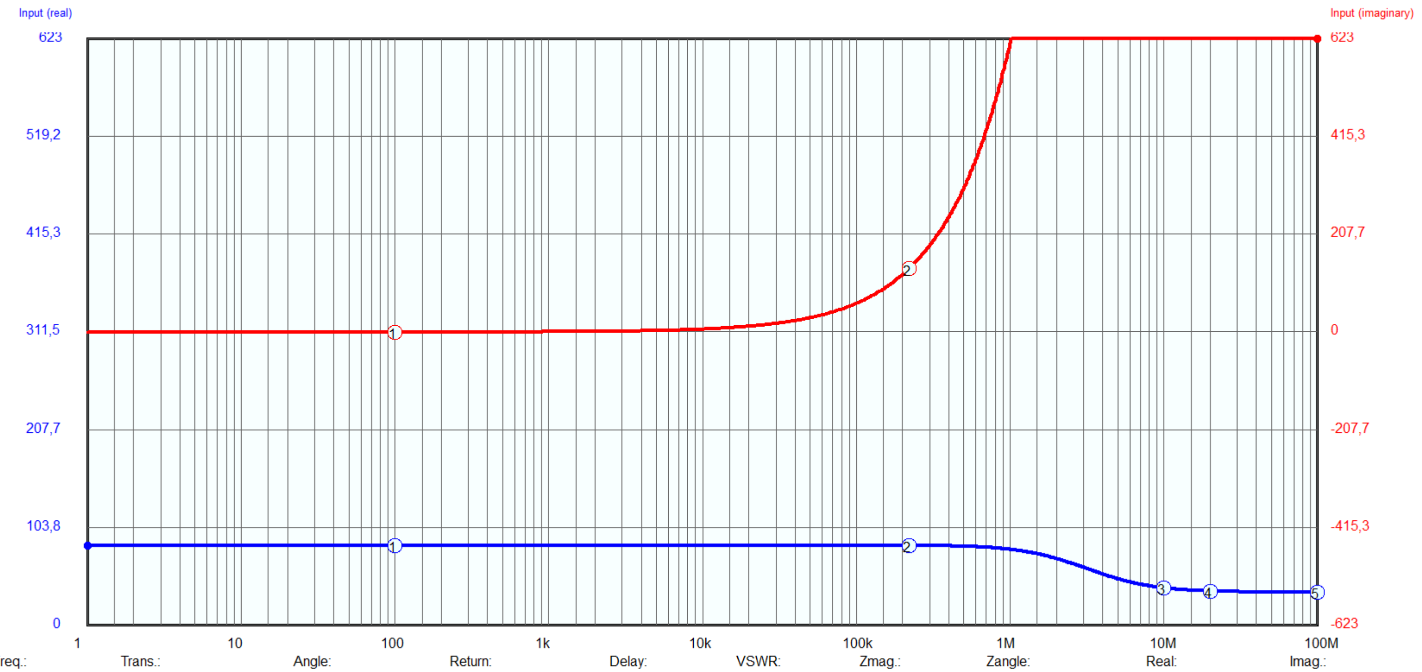
5	100M	-85,911	-178,07	0.0000	53,75p	>1000	62,83k	89,968	35,206	62,83k

Tiefpass 2. Ordnung



$$\underline{A}_T(\omega) = \frac{1}{1 - (\frac{\omega}{\omega_0})^2 + jD\,\frac{\omega}{\omega_0}}$$





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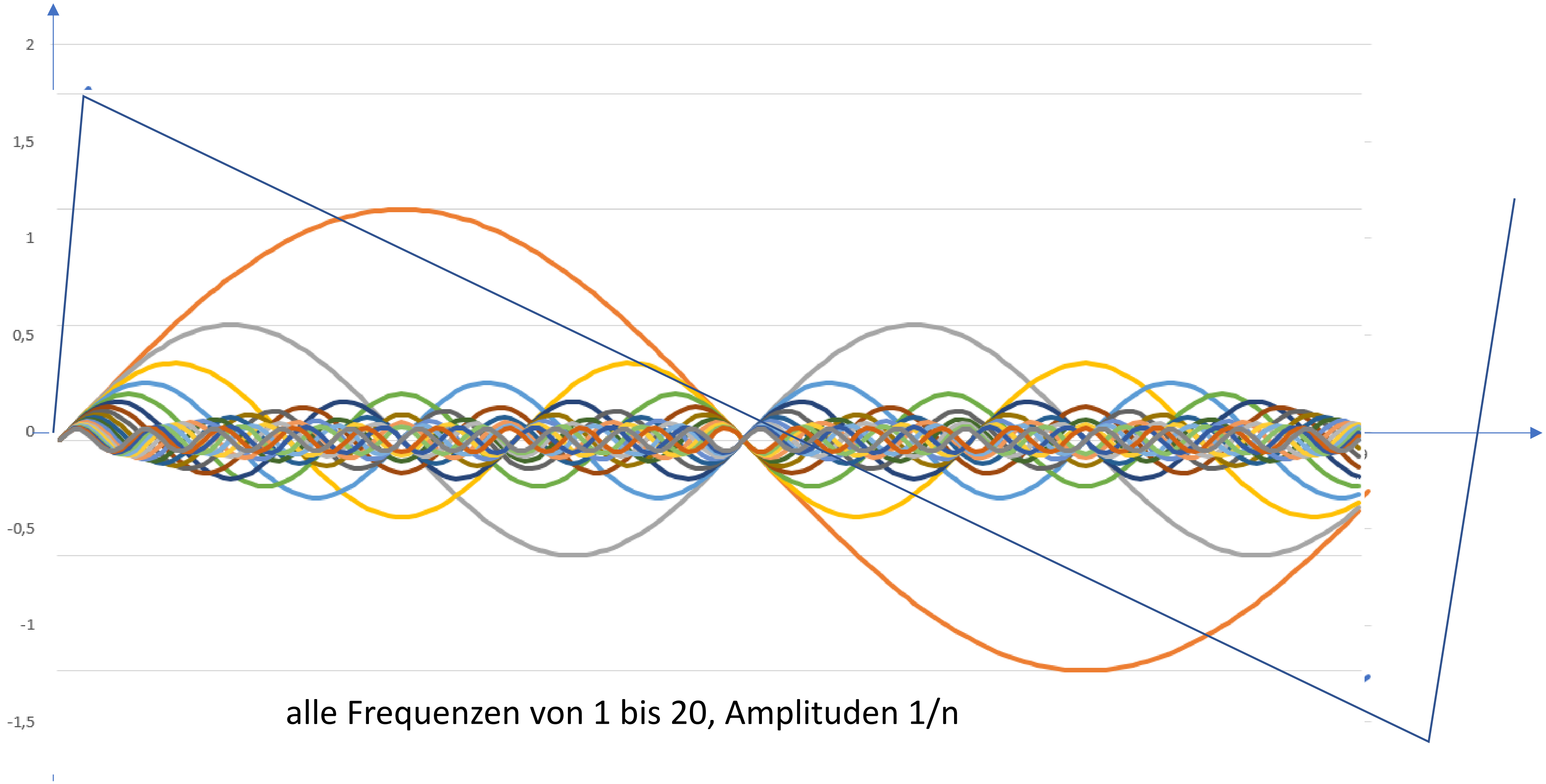
5	100M	-85,911	-178,07	0.0000	53,75p	>1000	62,83k	89,968	35,206	62,83k

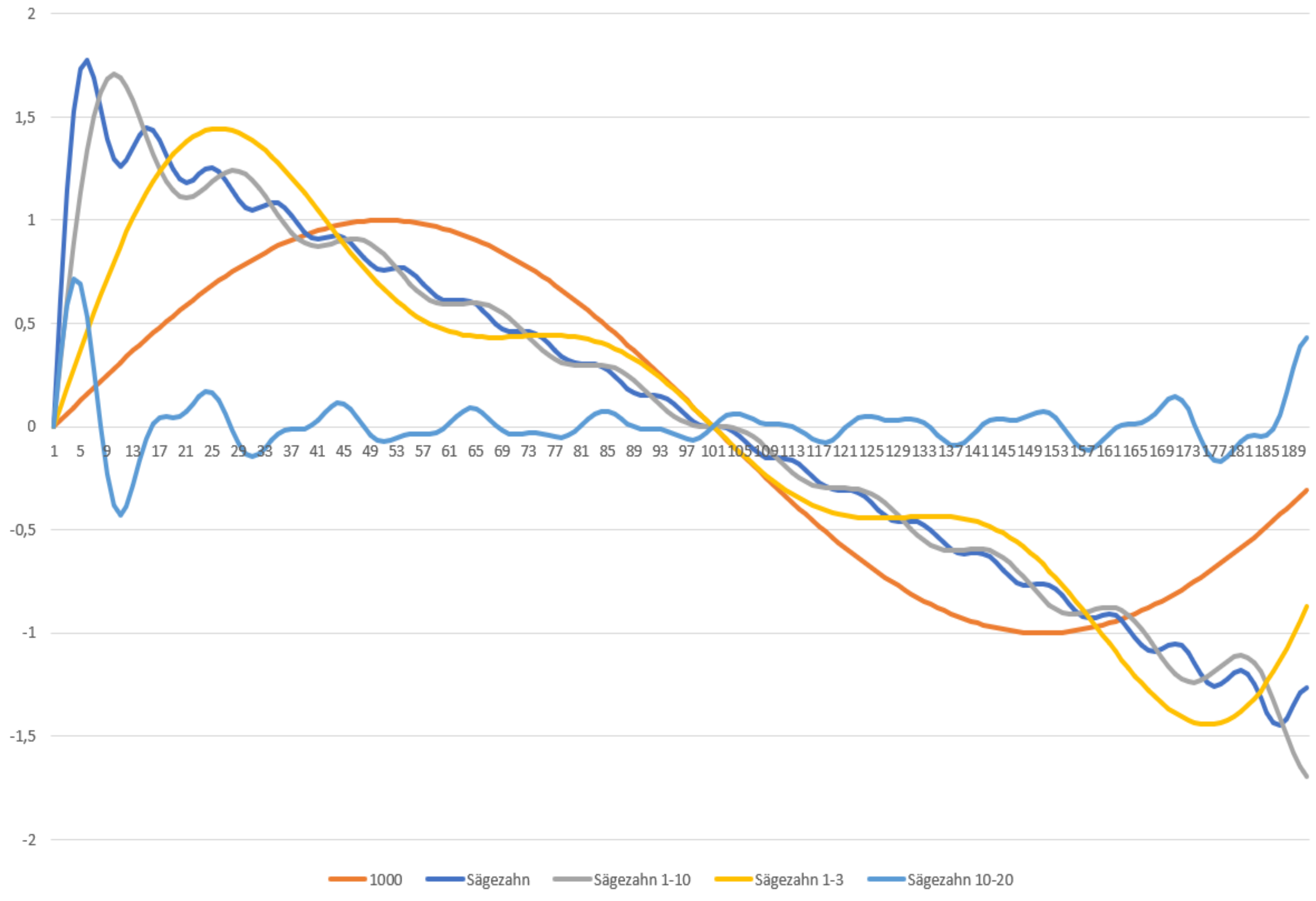
Anwendungsbeispiel

Synthese von Signalen, Sägezahn

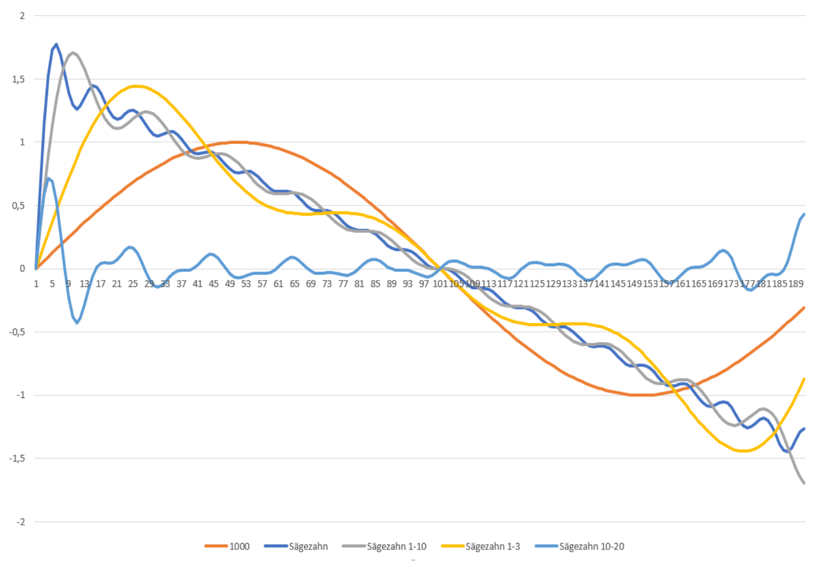


alle Frequenzen von 1 bis 20, Amplituden 1/n

Synthese von Signalen, Sägezahn



Synthese von Signalen, Sägezahn

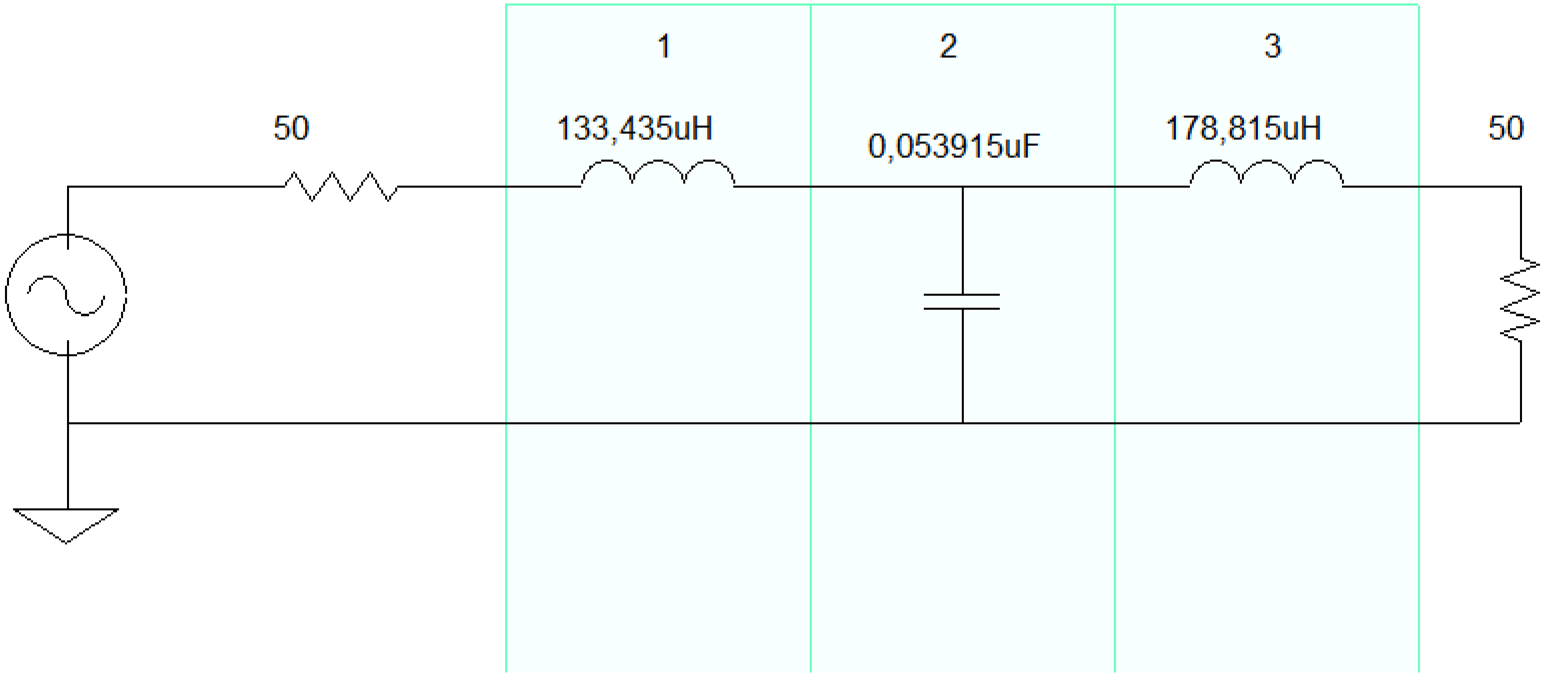


Annahme $f_{Sägezahn}$=10 kHz
1. – 10. Oberwelle : 20 kHz – 100 kHz
10. – 20. Oberwelle : 100 kHz – 200 kHz

Versuch 1 : Low-pass 1 – 100 kHz

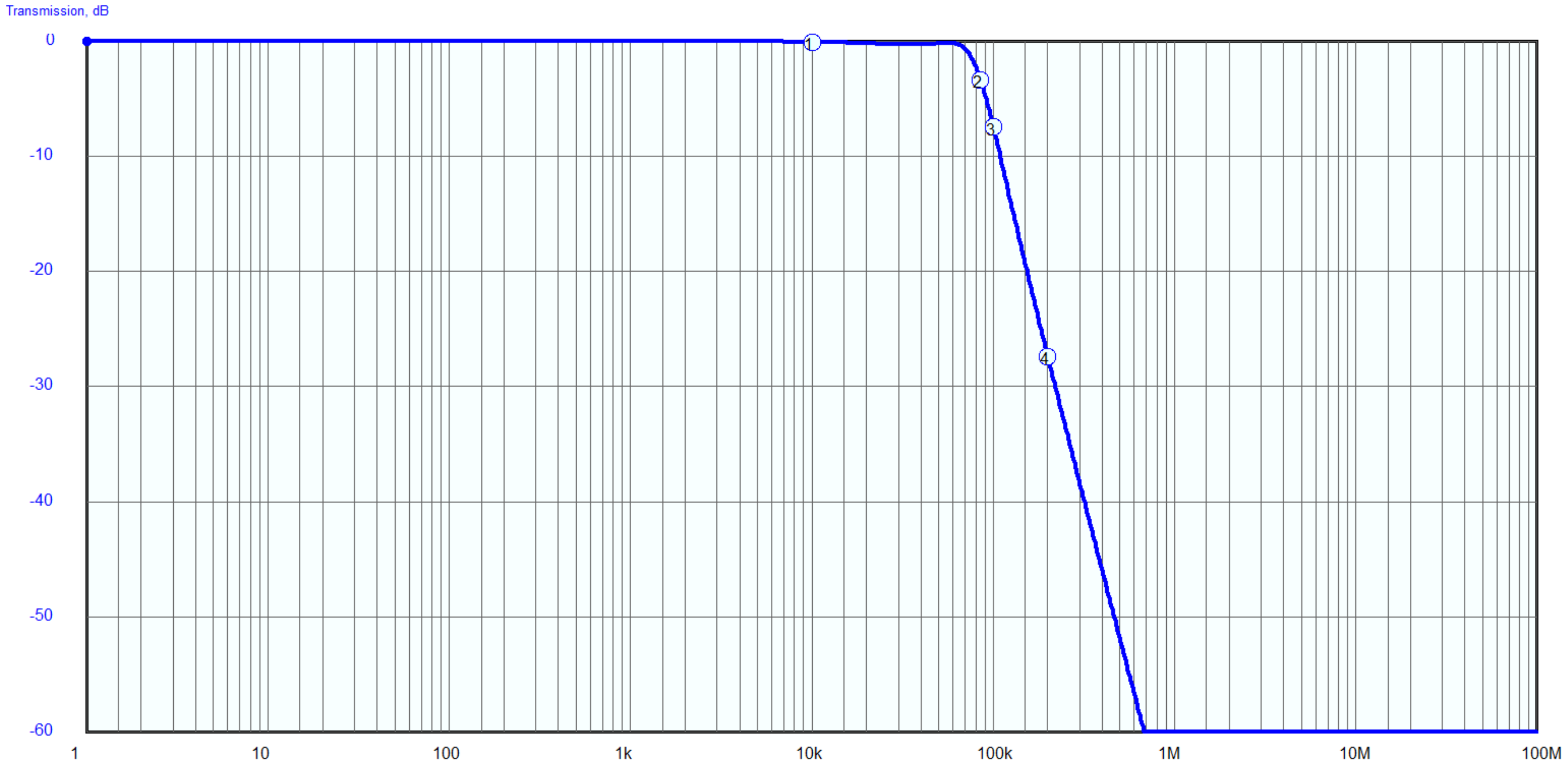
Versuch 2: high-pass 100 – 200 kHz

Versuch 1



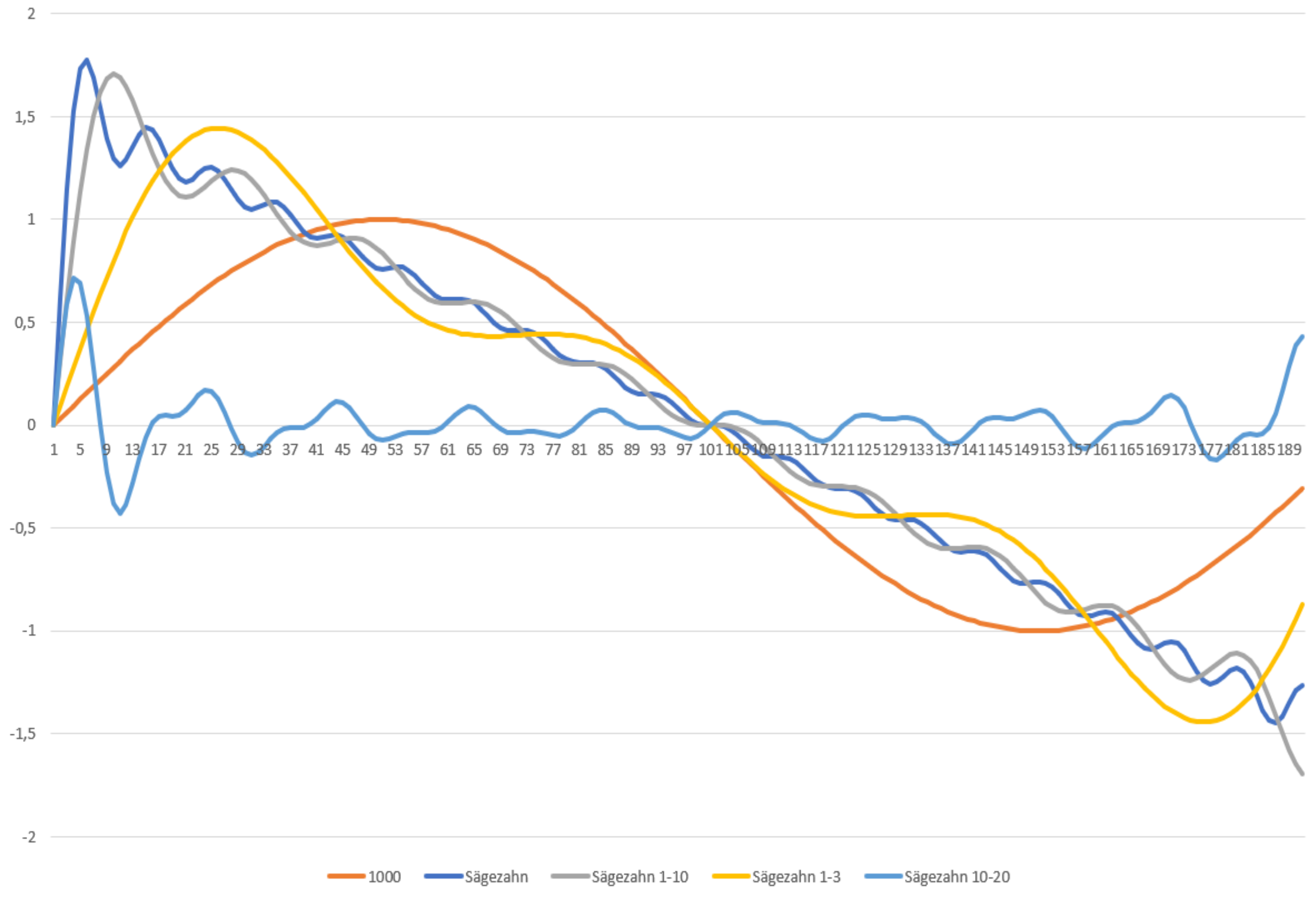
Low-pass Filter 1 – 100 kHz

Versuch 1

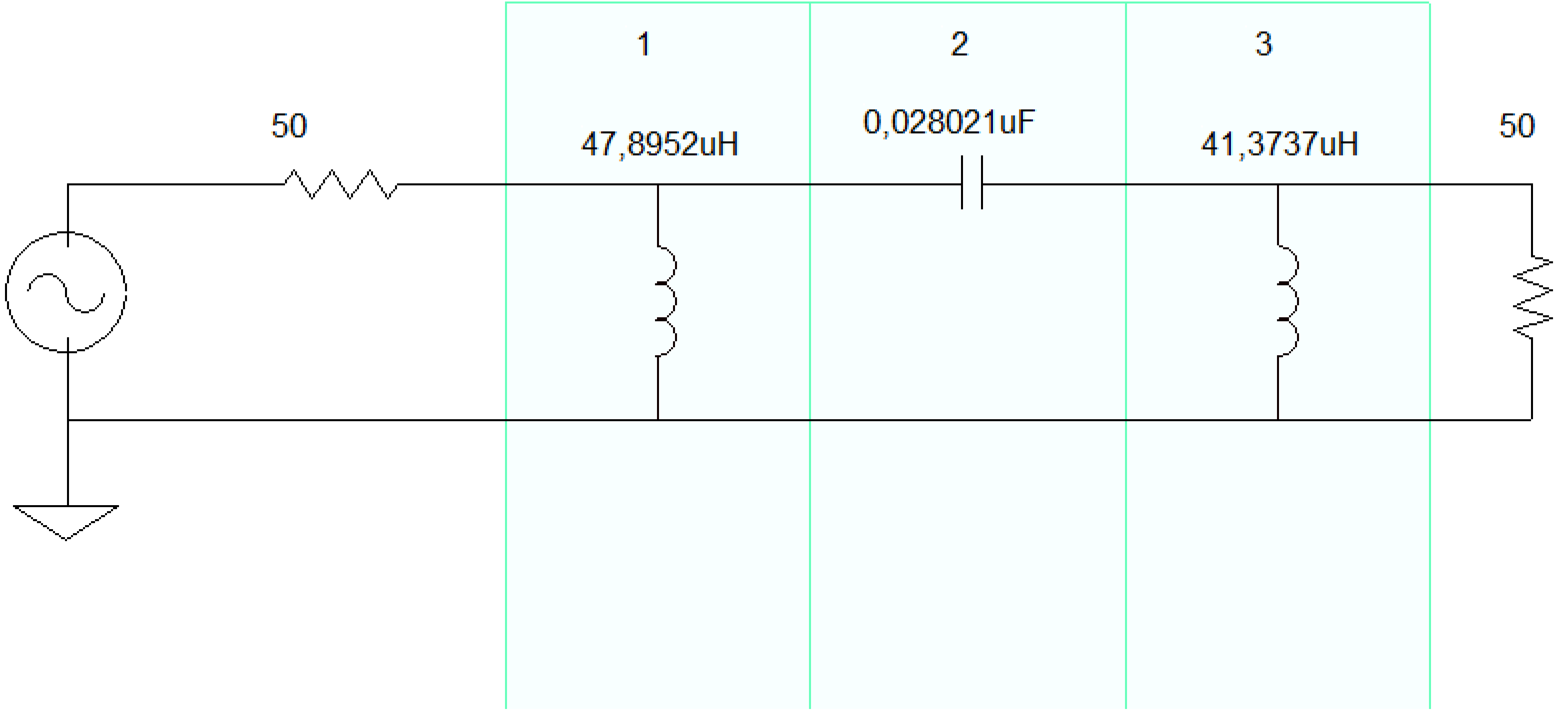


	Freq.:	Trans.:	Angle:	Return:	Delay:	VSWR:	Zmag.:	Zangle:	Real:	Imag.:
1	10k	-0,05154	-16,026	-19,35	4,417u	1,2416	53,518	11,679	52,41	10,834
2	85k	-3,3217	-162,98	-2,7335	6,178u	6,4076	26,574	68,43	9,7695	24,713
3	100k	-7,4409	170,71	-0,86865	3,9u	20,015	46,85	84,267	4,6796	46,615
4	200k	-27,384	121,73	-0,0083	510,1n	>1000	151,94	89,908	0,24436	151,94
5	1M	-70,049	96,009	0.0000	17n	>1000	835,44	90	0,0039	835,44

Synthese von Signalen, Sägezahn

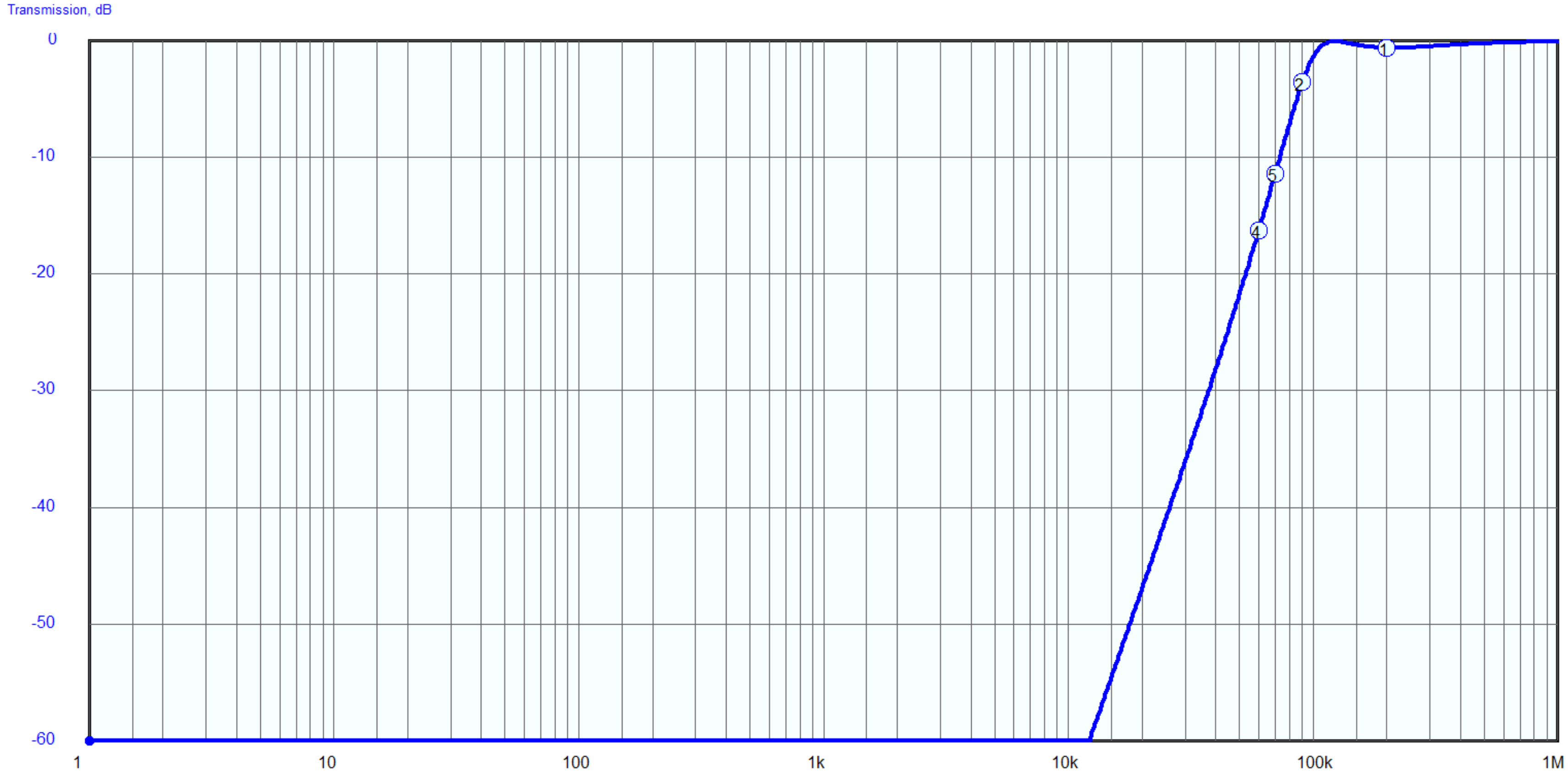


Versuch 2



High-pass Filter 100 – 1000 kHz

Versuch 2



	Freq.:	Trans.:	Angle:	Return:	Delay:	VSWR:	Zmag.:	Zangle:	Real:	Imag.:
1	200k	-0,6085	62,756	-8,8523	855,2n	2,1294	25,27	17,107	24,152	7,4332
2	90,3k	-3,5541	174,27	-2,54	6,102u	6,888	64,402	72,931	18,904	61,565
3	7k	-74,444	-94,578	-0,00026	1,779u	>1000	2,112	89,979	0,00076	2,112
4	60k	-16,198	-136,22	-0,10702	3,063u	162,32	23,257	89,077	0,37467	23,254
5	70k	-11,351	-148,73	-0,33328	3,87u	52,13	31	87,546	1,3273	30,972

Synthese von Signalen, Sägezahn